2016

MATHEMATICS II

(MATH 1201)

Time Alloted : 3 Hours *Full Marks : 70*

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : **[10×1=10]**

i) The equation $(3x^2 + py)dx + (-6y^2 + qx)dy = 0$ is exact if

(a) $p + q = 0$ (b) $p - q = 0$

(c) $3p + q = 0$ (d) $p \neq q$

ii) $\frac{1}{(D-1)^2}e^x =$

(a) e^x (b) xe^x

(c) x^2e^x (d) $\frac{x^2}{2}e^x$

iii) A tree T has two vertces of degree 2, one vertex of degree 3 and three vertices of degree 4. The number of pendant vertices in T is

(a) 9 (b) 6 (c) 7 (d) 10

iv) The number of vertices in a simple graph with 21 edges whose 3 vertices are of degree 4 and remaining vertices are of degree 3 is

(a) 10 (b) 13 (c) 12 (d) 20

v) A complete graph may not be

(a) regular (b) connected

(c) simple (d) circuit

vi) $L(e^{2t}) =$

(a) $\frac{1}{s-2}$ (b) $\frac{1}{s+2}$ (c) $\frac{1}{s^2}$ (d) $\frac{1}{2s}$

vii) $B\left(\frac{1}{2}, \frac{1}{2}\right) =$

(a) $\sqrt{\pi}$ (b) π (c) 1 (d) $\frac{1}{2}$

viii) The general solution of $p^2 + p - 6 = 0$ is $\left[\text{where, } p = \frac{dy}{dx}\right]$

(a) $(y + 3x - c)(y - 2x - c) = 0$

(b) $(y + 4x - c)(y - 2x - c) = 0$

(c) $(y + 3x)(y - 2x - c) = 0$

(d) none

ix) Determine the value of k so that the lines $\frac{x-1}{2} = \frac{y-4}{1}$ $= \frac{z-5}{2}$ and $\frac{x-2}{-1} = \frac{y-8}{k} = \frac{z-11}{4}$ may intersect

(a) 2 (b) 4

(c) 3 (d) 5

x) Angle between the planes $x + y + z = 1$, $x - y = 2$ is

(a) 0 (b) $\pi / 3$

(c) $\pi / 2$ (d) $\pi / 4$

GROUP - B

2. (a) Solve the following differential equation :

$$(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$$

(b) Obtain the differential equation of all circles each of which touches x-axis at the origin.

(c) Find the integrating factor of the following differential equation :

$$\frac{dx}{dy} + \frac{x}{y \log y} = \frac{2}{y}$$

6+4+2 = 12

3. (a) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + a^2y = \text{Sec } ax$$

(b) Find the general solution of the following equation by D-operator method:

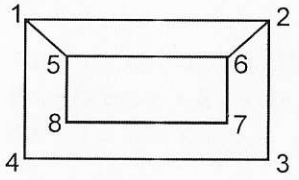
$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^{3x}$$

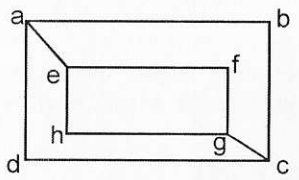
6+6 = 12

GROUP - C

4. (a) Let G be a graph every vertex of which is of even degree. Prove that G has no cut edge.

(b) Determine whether the following graphs are isomorphic or not :



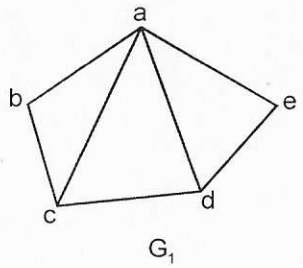


(c) Does there exist a graph corresponding to the following incidence matrix? Justify.

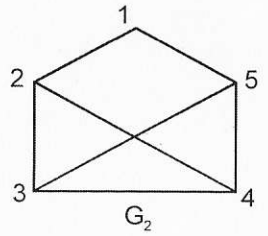
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6+4+2 = 12

5. (a) Find whether the following graphs are isomorphic or not. Give reasons for your answer.



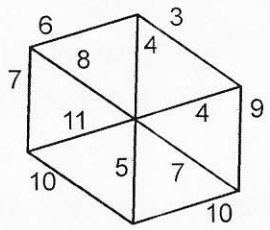
G_1



G_2

(b) Prove that every graph with fewer edges than vertices has a component that is a tree.

(c) Find by Prim's Algorithm, a minimal spanning tree of the following graph and find its weight.



3+5+4 = 12

Group - D

6. (a) Evaluate :

$$\int_0^{\infty} \frac{dx}{(1+x)\sqrt{x}}$$

(b) Prove that $\int_{-1}^{1} \frac{dx}{x^3}$ exists in Cauchy principal value sense but not in general sense.

6+6 = 12

7. (a) Evaluate the following using convolution theorem :

$$L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}.$$

(b) Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4e^{2t}$, $y(0) = -3, y'(0) = 5$

5+7 = 12

GROUP - E

8. (a) A variable plane has intercepts on the co-ordinate axes, the sum of whose squares is a constant k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is $(x^2 + y^2 + z^2) \times \left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = k^2$

(b) Find the equation of the bisector of the angle between the planes $x + 2y + 2z = 9$, $4x - 3y + 12z + 13 = 0$ that contains the origin.

6+6 = 12

9. (a) Find the magnitude and the equation of the line of shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}, \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

(b) Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$, $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. Find the equation of the plane on which they lie.

6+6 = 12